PARAMETERIZATION OF DRY DEPOSITION PROCESSES IN THE SURFACE LAYER FOR ADMIXTURES WITH GRAVITY DEPOSITION

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Outline:
The present paper aims are:
➢ To suggest a more general treatment of the aerodynamic resistance;
➢ To demonstrate the significance of the proposed parameterization.
\[-F = k \frac{dc}{dz} = V_d(z)(c(z) - c_s), \quad (1)\]

where \(c(z)\) is the admixture concentration, \(c_s\) is the admixture concentration at the absorbing surface (as it is unknown it is usually assumed \(c_s \sim 0\)), \(c_s\) \(V_d\) is the dry deposition velocity and \(k(z)\) is the coefficient of vertical turbulent exchange.

By making analogy with the Ohms law in electrical circuits the dry deposition velocity \(V_d\) is most often presented in the form:

\[V_d = (r_a + r_b + r_s)^{-1}, \quad (2)\]

where \(r_a\) is the surface layer (SL) aerodynamic resistance, \(r_b\) is the quasi-laminar or viscous sub-layer resistance, \(r_b\) \(r_s\) is the surface resistance.
It seems that the most general expression for the aerodynamic resistance $r_a$ is the following:

$$r_a(z) = \int_{z_0}^{z} \frac{dz}{k(z)}, \quad (3)$$

where $z_0$ is the roughness length.

If it is assumed, as usual, that the SL turbulent transport of admixtures is similar to these of heat and momentum, it can be written:

$$k(z) = \frac{\kappa u_* z}{\varphi(\zeta)}, \quad (4)$$

where $\kappa$ - the von Karman constant, $u_*$ - friction velocity, $\varphi(\zeta)$ - the universal function of the dimensionless height $\zeta = z / L$, $L$ - the Monin-Obuchov length.

In such a case the expression for $r_a$ resumes the form:

$$r_a(z) = \frac{1}{\kappa u_*} \left( f(z) - f(z_0) \right), \quad f(z) = \int \frac{\varphi(\zeta)}{z} dz. \quad (5)$$
From (3) it is obvious that $r_a(z_0) = 0$ and thus

$$V_{d0} = V_d(z_0) = \left( r_b + r_s \right)^{-1}, \quad (6)$$

i.e. the deposition velocity at roughness length height is subject only of the transport of the component through the laminar layer adjacent to the surface by molecular diffusion and the various destruction or uptake processes of the component at the surface.

The vertical profile $c(z)$ of the concentration of an admixture with gravity deposition $-w_g, (w_g > 0)$ is described by the equation:

$$\frac{d}{dz} k \frac{dc}{dz} + w_g \frac{dc}{dz} = q \delta(z - z_{source}), \quad (7)$$

where $q$ is the capacity of a flat (locally) homogeneous admixture source, $\delta$ is the Dirac function.

The boundary condition at $z = z_0$ is, according to (1), (2), (6), the following:

$$k \frac{dc}{dz} = V_{a0} c_0, \quad (8)$$

c_0 - the concentration at $z = z_0$. 

The integration of (7), having in mind also (8) leads to:

\[ \frac{kd}{dz} + w_g c = \left( w_g + V_{d0} \right) c_0 - q H_{\text{source}}(z), \quad (9) \]

where \( H_{\text{source}}(z) \) is the Heaviside function \( (H_{\text{source}}(z) = 0 \text{ for } z < z_{\text{source}}; \quad H_{\text{source}}(z) = 1 \text{ for } z > z_{\text{source}}) \).

By the transformation

\[ c = x e^{-w_g r_a}, \quad (10) \]

where \( r_a \) is the aerodynamic resistance (see (3)), equation (9) can be simplified to the form (from (10) it is obvious that \( x(z_0) = c_0 \)):

\[ \frac{kdx}{dz} = \left( w_g + V_{d0} \right) c_0 e^{w_g r_a} - q H_{\text{source}}(z) e^{w_g r_a}, \quad (11) \]
After some trivial manipulations an expression for $c(z)$ to be obtained:

$$
c(z) = \left[1 + \frac{V_{d0}}{w_g} \left(1 - e^{-w_g r_a(z)}\right)\right] c_0 - H_{\text{source}}(z) \frac{q}{w_g} \left(1 - e^{w_g r_a(z_{\text{source}}) - w_g r_a(z)}\right) \tag{12}
$$

The flux/concentration relation for the case of admixtures with gravity deposition and possible sources in the SL:

$$
k \frac{dc}{dz} = V_d(z) c(z) - H_{\text{source}}(z) \frac{V_d(z)}{V_d(z_{\text{source}})} q, \quad \tag{13}
$$

where

$$
V_d(z) = \left[\frac{1}{w_g} \left(e^{w_g r_a(z)} - 1\right) + e^{w_g r_a(z)} \frac{1}{V_{d0}}\right]^{-1}, \quad \tag{14}
$$

It is easy to calculate that in case of admixture with no gravity deposition ($w_g \to 0$) the expression (14) takes the form (2). Further, if there are no sources in the SL the flux/concentration relation transforms into the form (1).
The particular cases when $V_{d0} \to \infty$ (total absorption at $z=z_0$) and $V_{d0} \to 0$ (total reflection at $z=z_0$): In the first case

$$V_d(z) \to \left[ \frac{1}{w_g} \left( e^{w_g r_a(z)} - 1 \right) \right]^{-1}, \text{ when } V_{d0} \to \infty, \quad (15)$$

and, as it can be easily seen from (13), there will be zero concentration at $z = z_0$. In the second case $V_d \to 0$ when $V_{d0} \to 0$, but the ratio $V_d(z) / V_d(z_{source})$ remains limited, so the relation (13) obtains the form:

$$k \frac{dc}{dz} = - H_{source}(z) \frac{e^{w_g r_a(z_{source})}}{e^{w_g r_a(z)}} q \ , \quad (16)$$

or in the case with no gravity deposition ($w_g \to 0$):

$$k \frac{dc}{dz} = - H_{source}(z) q \ . \quad (17)$$
SOME EXAMPLES
If the deposition velocity, calculated according to (1) is denoted by $V_{d1}$, then having in mind (6) the aerodynamic resistance $r_a$ may be expressed in the form:

$$r_a = V_{d1}^{-1} - V_{d0}^{-1}, \quad (19)$$

Inserting (19) in (15), leads, after some simple transformations to the dimensionless relation:

$$\tilde{\nu}_d = \left[ \frac{1}{\tilde{w}_g} \left( e^{\tilde{w}_g (\tilde{\nu}_{d1}^{-1} - 1)} + e^{\tilde{w}_g (\tilde{\nu}_{d1}^{-1} - 1)} \right) \right]^{-1}, \quad (19)$$

where $\tilde{\nu}_d = V_d / V_{d0}$, $\tilde{\nu}_{d1} = V_{d1} / V_{d0}$, $\tilde{w}_g = w_g / V_{d0}$. 
Figure 1. The difference between $\tilde{V}_d$ and $\tilde{V}_{d1}$ for different $\tilde{w}_g$ values: $\tilde{w}_g = 0$ (0), 0.05 (1), 0.1 (2), 0.5 (3), 1 (4), 3 (5), 10 (6), 50 (7) and 100 (8)
Let a two-layer model for $k$ is assumed – $k = k(z)$, calculated according to (4) in the SL ($z_0 \leq z \leq h_{SL}$), $h_{SL}$ - the SL height; $k = k_h = k(h_{SL})$ for $h_{SL} \leq z < \infty$.

Then, in the horizontally homogeneous case, the vertical profile above SL of the concentration $c(z, t)$ from an instantaneous flat source with height $h > h_{SL}$ can be obtained from the equation:

$$\frac{\partial c}{\partial t} - w_g \frac{\partial c}{\partial z} + k_h \frac{\partial^2 c}{\partial z^2} = 0, \quad h_{SL} \leq z < \infty, \quad (20)$$

under the following initial and boundary conditions:

$$c(z, 0) = \delta(z - h); \quad k_h \frac{\partial c}{\partial z} = \beta e(h_{SL}, t). \quad (21)$$

Here $\beta$ is the dry deposition velocity at $z = h_{SL}$. Depending on the chosen parameterization $\beta$ is equal to $V_{d}(z = h_{SL})$ or to $V_{d1}(z = h_{SL})$ - respectively the cases when the gravity deposition effects on the aerodynamic resistance are accounted, or not accounted for.
As it can be easily shown (Galperin M., D.L. Yordanov and K.G. Ganev, 2000), the solution of (20-21) is:

$$c(z, t) = e^{-\frac{w_g (z'-h')}{2k_h} - \frac{w_g^2 t}{4k_h}} \left\{ \frac{1}{2\sqrt{\pi k_h t}} e^{-\frac{(z'-h')^2}{4k_h t}} + e^{-\frac{(z+h)^2}{4k_h t}} \right\} - \frac{\beta}{k_h} e^{\frac{\beta (z'+h'+\tilde{\beta} t)}{k_h}} \text{erfc}\left(\frac{z'+h'+2\tilde{\beta} t}{2\sqrt{k_h t}}\right),$$

where $\tilde{\beta} = \beta + \frac{w_g}{2}$, $z' = z - h_{SL}$ and $h' = h - h_{SL}$.

This formula was applied for calculating the concentrations $c'(h_{SL}, t)$ and $c''(h_{SL}, t)$ at SL height for the cases when gravity deposition is accounted ($\beta = V_d(z = h_{SL})$) and not accounted ($\beta = V_d(z = h_{SL})$) for. The calculations were made for a wide range of $V_d$ and $w_g$ values for the cases of stable ($u_* = 0.5 \text{ m/s}$, $L=10$), neutral ($u_* = 0.2 \text{ m/s}$) and unstable ($u_* = 0.2 \text{ m/s}$, $L=-10$) stratification for source height $h=200\text{m}$. The corresponding maximal concentration values $c'_{\text{max}}(V_d, w_g)$, $c''_{\text{max}}(V_d, w_g)$ and their relative difference $D(V_d, w_g) = (c'_{\text{max}} - c''_{\text{max}})/c'_{\text{max}}$ are determined for all the stability cases.
Figure 2. Dependence of $D$ on $w_g$ and $V_{d0}$ - $V_{d0} = 10^{-3} \text{m/s}$ (1); $10^{-2} \text{m/s}$ (2); $10^{-1} \text{m/s}$ (3); 1m/s (4) for the cases of stable (a), neutral (b) and unstable (c) stratification.

1) gravity deposition may have a significant effect in stable and neutral cases;
2) the effect is maximal for gravity deposition velocities between 0.01 and 0.1 $\text{ms}^{-1}$;
3) the effect increases with the increasing of $V_{d0}$ up to values of 0.1 $\text{ms}^{-1}$, after which the value of the relative difference $D$ does not change much.
CONCLUSIONS

➢ The most popular parameterization schemes treat the aerodynamic resistance and the gravity deposition independently, most often by simply adding the gravity deposition velocity. As the gravity deposition modifies the admixture profiles and thus the admixture turbulent fluxes in the SL, this approach is obviously incorrect.

➢ The present paper suggests a more general approach, based on the exact solution of the pollution transport (turbulent and gravity deposition) equation in the SL, which provides a correct expression for the aerodynamic resistance, accounting also for the gravity deposition effects.

➢ The demonstrated examples show the importance of a joint treatment of turbulent transport and gravity deposition in calculating the aerodynamic resistance.
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REFERENCES


