4.07 STRATIFICATION, BAROCLINICITY AND INVERSION EFFECTS ON THE
LAGRANGIAN AND DIFFUSION CHARACTERISTICS OF INSTANTANEOUSLY
RELEASED CLOUD IN THE PBL

Evgeni Syrakov\(^1\) and Kostadin Ganev\(^2\)
\(^1\)Faculty of Physics, University of Sofia, Sofia, Bulgaria
\(^2\)Institute of Geophysics, Bulgarian Academy of Sciences, Sofia, Bulgaria

INTRODUCTION

The shear-generated effects on the centroids and dispersions of an instantaneously released cloud had been studied in Safman P., 1962, Smith F., 1965. Similar effects obtained by an Ekman’s PBL model are demonstrated in Csanady G., 1969. Different modifications and applications can be seen in a number of following papers (for example Luhar A., 2002). The present work aims at further study of these phenomena by relatively general accounting for effects of none-stationarity, stratification, baroclinicity, inversions and terrain slopes.

MODEL DESCRIPTION

The following well-known system of equations, based on the method of the moments is applied:

\[
\begin{align*}
\frac{\partial c_{00}}{\partial t} + (w - w_0) \frac{\partial c_{00}}{\partial z} + \ddot{c}_0 &= \frac{\partial}{\partial z} k_x \frac{\partial c_{00}}{\partial z} \\
\frac{\partial c_{10}}{\partial t} + (w - w_0) \frac{\partial c_{10}}{\partial z} + \ddot{c}_1 &= \frac{\partial}{\partial z} k_z \frac{\partial c_{10}}{\partial z} + uc_{00} \\
\frac{\partial c_{20}}{\partial t} + (w - w_0) \frac{\partial c_{20}}{\partial z} + \ddot{c}_2 &= \frac{\partial}{\partial z} k_z \frac{\partial c_{20}}{\partial z} + 2k_x c_{00} + 2uc_{10}
\end{align*}
\]

The corresponding boundary and initial conditions for unity power source are:

\[
\begin{align*}
k_z \frac{\partial \ddot{c}}{\partial z} &= (B - w_0) \dot{c} \text{ at } z = z_0 \quad , \quad k_z \frac{\partial \ddot{c}}{\partial z} = 0 \text{ at } z = H_T \quad , \quad \dot{c} = c_{00}, c_{10}, c_{20} \\
 c_{00} &= \delta(z - h) \quad , \quad c_{10} = c_{20} = 0 \text{ at } t = 0
\end{align*}
\]

where \( c_{00}(z,t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(x,y,z,t) dx dy \) is the zero moment, \( c_{10}(z,t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(x,y,z,t) x dx dy \) and \( c_{20}(z,t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(x,y,z,t) x^2 dx dy \) are the first and the second moments in respect to the \( x \) axis.

The equations and the initial and boundary conditions for the corresponding first and the second moments in respect to the \( y \) axis \( c_{01}(z,t) \) and \( c_{02}(z,t) \) are similar, with \( v \) and \( k_y \) staying in the place of \( u \) and \( k_x \). The other qualities are denoted in the following way: \( u(z) \), \( v(z) \) and \( w(z) \) - the wind components; \( H_T \) - diffusion layer height; \( w_0 \), \( B \), \( \ddot{c} \) - gravity deposition velocity, deposition parameter, describing the admixture-soil interaction and the chemical transformation parameter; \( k_z \), \( k_x \), \( k_y \) - the vertical and horizontal turbulent exchange coefficients; \( h \) - the effective stack height. The horizontal and vertical cloud characteristics can be expressed in the form:

\[
X = c_{10} / c_{00} \quad , \quad \sigma_x = \sqrt{c_{20} / c_{00} - X^2} \quad , \quad Y = c_{01} / c_{00} \quad , \quad \sigma_y = \sqrt{c_{02} / c_{00} - Y^2}
\]
\[ Z = \int_{0}^{\infty} c_{00} z \, dz / c_{0}, \quad \sigma_{z} = \sqrt{\left( \int_{0}^{\infty} c_{00} z^{2} \, dz / c_{0} - Z^{2} \right)}, \quad Sk = \int_{0}^{\infty} c_{00} (z - Z)^{3} \, dz / (c_{0} \sigma_{z}^{3}), \]

\[ Ku = \int_{0}^{\infty} c_{00} (z - Z)^{4} \, dz / (c_{0} \sigma_{z}^{4}) \]

where \( X, Y, Z \) are the centroid coordinates, \( \sigma_{x}, \sigma_{y}, \sigma_{z} \) are the cloud dispersions, \( Sk \) is the skewness and \( Ku \) – the kurtosis.

The dynamic parameters \( u, v, k_{z} \) \((k_{x} = k_{y} = p k_{z}, p = 5)\) in (1)-(5) are determined by a model of a none-stationary, stratified, baroclinic, inversion dependent PBL over a sloping terrain (Syrakov E. and K. Ganev, 2002): The problem is generally enough parameterized by a set of external dimensionless parameters (Syrakov E., 1990, 2002):

\[ Ro(t), S(t); Ro_{i}; S_{x}, S_{y}, (or M, \Psi); \phi, \psi, \]

where \( Ro = G_{0} / f z_{0}, Ro_{i} = G_{0} / f H_{i}, S = \beta \delta 9 / G_{0} f \) are the geostrophic and inversion Rosby number and the external stratification parameter, \( S_{x} = (\kappa^{2} / f) du_{g} / dz \), \( S_{y} = (\kappa^{2} / f) dv_{g} / dz \) - baroclinic parameters (Wippermann, F. 1972), \( M = (S_{x}^{2} + S_{y}^{2})^{1/2}, \Phi \) - the angle between the surface geostrophic and the thermal wind, \( \phi, \psi \) - the slope angles in \( x \) and \( y \) directions.

**RESULTS AND DISCUSSION**

The model (1)-(7), applied for the calculations demonstrated below at \( B=\tilde{a}=w_{0} = 0 \), was validated by comparison with analytic solutions given by Csanady G., 1969. The \( Ox \) axis is always oriented along the surface geostrophic wind. A set of typical situations is considered in this study. Three cases are chosen as basic ones: unstable \((S=-500)\), neutral \((S=0)\) and stable \((S=500)\) at \( \log(Ro) = 7 \). Calculations are made also for \( \log(Ro)=5 \), which outlines the influence of the roughness length \( z_{o} \). All the other calculations are made for \( \log(Ro)=7 \) (Table 1). Two cases of low inversions are calculated for unstable and neutral stratifications (Table 1). The baroclinic parameters are varied for the unstable basic case at \( M = 10 \) at \( \Phi=0(barocl.1), 180(barocl.2), 220(barocl.3), 270(barocl.4) \). The slope effects \((\phi=0, \psi = 0.1rad)\) are studied for the unstable and stable basic cases and \( Ox \) up-slope oriented, the free atmosphere temperature gradient \( \Gamma = 0.006 \text{deg/m} \). In both the cases the geostrophic wind is up-slope oriented. It was assumed \( H_{T} = H_{i} \) in the inversion and \( H_{T} = H_{PBL} \) in the other cases.

**Table 1. Input and some calculated (drag coefficient Cd, full wind rotation angle \( \alpha \) and internal stratification parameter \( \mu \)) PBL parameters**

<table>
<thead>
<tr>
<th>Case</th>
<th>( S )</th>
<th>( G_{0} )</th>
<th>( Ro )</th>
<th>( Ro_{i} )</th>
<th>( H_{PBL}/H_{i} )</th>
<th>( Cd )</th>
<th>( \alpha )</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stable</td>
<td>500</td>
<td>8m/s</td>
<td>10⁷</td>
<td>-</td>
<td>350m</td>
<td>0.016</td>
<td>33°</td>
<td>45</td>
</tr>
<tr>
<td>Neutral</td>
<td>0</td>
<td>8m/s</td>
<td>10⁷</td>
<td>-</td>
<td>850m</td>
<td>0.029</td>
<td>23°</td>
<td>0</td>
</tr>
<tr>
<td>Unstable</td>
<td>-500</td>
<td>8m/s</td>
<td>10⁷</td>
<td>-</td>
<td>1400m</td>
<td>0.048</td>
<td>8°</td>
<td>-100</td>
</tr>
<tr>
<td>Neutral inv.2</td>
<td>0</td>
<td>8m/s</td>
<td>10⁷</td>
<td>400</td>
<td>200m</td>
<td>0.025</td>
<td>35°</td>
<td>0</td>
</tr>
<tr>
<td>Unstable inv.1</td>
<td>-500</td>
<td>8m/s</td>
<td>10⁷</td>
<td>200</td>
<td>400m</td>
<td>0.042</td>
<td>30°</td>
<td>-150</td>
</tr>
<tr>
<td>Unstable inv.2</td>
<td>-500</td>
<td>8m/s</td>
<td>10⁷</td>
<td>400</td>
<td>200m</td>
<td>0.034</td>
<td>44°</td>
<td>-260</td>
</tr>
</tbody>
</table>
The present work focuses mainly on the study of the horizontal cloud characteristics, but some PBL effects on the vertical characteristics like $\sigma_z$, $Sk$ and $Ku$ will also be demonstrated - see Fig.1. It can be seen that $\sigma_z$ increases with the increase of roughness length and $h$. The factors influence of $h$ on $Sk$ and $Ku$ is maximal in the stable case.

The horizontal cloud characteristics will be discussed below. The surface centroid trajectory analysis (Fig.2) shows that the trajectory declination from the geostrophic wind increases with the stability and roughness increase (Fig.2.a). The declination from the basic cases, caused by inversions, is significant only in the neutral case (Fig.2.b). As it should be expected, the largest trajectory deviations are observed in the baroclinic cases (Fig.2.c). The terrain slope effects on the trajectory declination are most significant for the stable case, because for the chosen experiment the geostrophic and the slope wind have opposite orientation (Fig.2.d).

The juxtaposition of Figures 3 and 4 shows, that for all the cases $\sigma_x(t) > \sigma_y(t)$, i.e. the gradient and other PBL factors are a stronger forcing factor along the surface geostrophic wind. The stability and roughness influences on $\sigma_x$ and $\sigma_y$ are qualitatively similar – increase with the roughness length and stability growth (Figs.3.a, 4.a). The maximal shear effect is observed in $\sigma_x(t)$, stable case, log($Ro$)=5. The low inversions influence is towards decreasing of $\sigma_x$ and $\sigma_y$ and towards the trend of $\sqrt{t}$ at large $t$, which is typical for fully-bounded atmosphere (Figs.3b, 4.b).

The baroclinicity influences on the horizontal dispersions are very complex and displayed in different ways for $\sigma_x(t)$ (Fig.3.c) and $\sigma_y(t)$ (Fig.4.c). The analysis shows that in $y$ direction the net “equivalent” shear effect (formed by barotropic-baroclinic and other PBL factors) surpasses the pure barotropic one, and hence the baroclinic $\sigma_y(t)$ are larger than the barotropic one for all the considered cases 1-4. If $\sigma_x(t)$ is considered the joint barotropic-baroclinic $x$-aligned shear effect is maximal (a superposition of the geostrophic and thermal winds) at $\Phi=0$ and hence in this case $\sigma_x(t)$ significantly surpasses the barotropic one. For all the other baroclinic cases 2-4 the joint barotropic-baroclinic shear effects, as had been underlined above, are manifested mainly in lateral direction, and so $\sigma_x(t)$ are smaller in comparison with the barotropic case (Fig.4.c).
Figure 2. Plots of $Y(X)$ at $z=0$, demonstrating the effects of roughness length (a), inversions (b), baroclinicity (c) and slopes (d) for low stack height ($h=5m$).

Figure 3. Plots of $\sigma_x(t)$, demonstrating the effects of stratification and roughness (a), inversions (b), baroclinicity (c) and slopes (d). The curves are denoted as in Fig.2.

The geostrophic and slope winds are in the same direction in the unstable case and that is why they generate maximal shear along the slope, which results in the fact that $\sigma_x(t)$ is significantly larger than the basic case (Fig.3.d). $\sigma_y(t)$ is also larger but to a smaller degree (Fig.4.d). In the stable case the geostrophic and the slope wind are oppositely directed along the $0x$ axis. As a result $\sigma_x(t)$ is also larger than the corresponding basic case (Fig.3.d), while for the lateral dispersion $\sigma_y(t)$ a contrary effect is observed (Fig.4.d).
Figure 4. Plots of $\sigma_y(t)$, demonstrating the effects of stratification and roughness (a), inversions (b), baroclinicity (c) and slopes (d). The curves are denoted as in Fig.2.

CONCLUSION

The examples demonstrated above show that the cloud characteristics (trajectories, dispersions, etc.) depend on the PBL dynamics in a very complex way. That is why it is practically impossible to introduce some universal relations. That is why it seems reasonable to use the well-known statistically-based construction, which divides the vertical $c_{v0}$ and horizontal $c_{hor}$ diffusion in the model of instantaneous cloud:

$$c(x, y, z, t) = c_{v0}(z, t) c_{hor}, \quad c_{hor} = \frac{1}{2 \pi \sigma_x \sigma_y} \exp\left(-\frac{(x-X)^2}{2 \sigma_x^2} - \frac{(y-Y)^2}{2 \sigma_y^2}\right), \quad (9)$$

to be applied jointly with the system (1)-(8). The approach can be also applied for studying some more specific cases like still, fumigation, PBL evolution as well as for regional and large-scale studies (see Smith, F., 1965). In the last case a two-layer model should be constructed, taking into account the PBL-free atmosphere interactions.

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